Chapter 2 of Gali (2008)

Introduction

Now, the dominant analytical framework to study monetary policy is the New Keynesian (NK) model (presented in Gali, Chapter 3). Its elements are: monopolistic competition and price stickiness. Before going into the NK model, we will study a classical monetary model featuring: perfect competition and fully flexible prices.

As stressed below, many of the predictions of that classical economy are strongly at odds with empirical evidence. That notwithstanding, the analysis of the classical economy provides a benchmark that will be useful in subsequent notes when some of its strong assumptions are relaxed. It also allows for the introduction of some notation, as well as assumptions on preferences and technology that are used in the remainder of our discussion.

In this economy, there are households (consume, supply labor); firms (produce, hire labor); the central bank (sets nominal interest rates and/or the money supply). Following much of the recent literature, the baseline classical model developed here attaches a very limited role to money. This is a cashless economy, so the only explicit role played by money is to serve as a unit of account.

Households

The representative household seeks to maximize the objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where $C_t$ is the quantity consumed of the single good, and $N_t$ denotes hours of work or employment. (Note that $N_t$ can be interpreted as the number of household members employed, assuming a large household and ignoring integer constraints). The period utility $U(C_t, N_t)$ is assumed to be
continuous and twice differentiable, with
\[ U_{c,t} = \frac{\partial U(C_t, N_t)}{\partial C_t} > 0, \quad U_{cc,t} = \frac{\partial^2 U(C_t, N_t)}{\partial C_t^2} \leq 0 \]
\[ U_{n,t} = \frac{\partial U(C_t, N_t)}{\partial N_t} \leq 0, \quad U_{nn,t} = \frac{\partial^2 U(C_t, N_t)}{\partial N_t^2} \leq 0 \]

In words, the marginal utility of consumption \( U_{c,t} \) is assumed to be positive and nonincreasing, while the marginal disutility of labor, \(-U_{n,t}\), is positive and nondecreasing.

Maximization of (1) is subject to a sequence of flow budget constraints given by
\[ P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t; \quad \forall \ t = 0, 1, \ldots \tag{2} \]

\( P_t \) is the price of the consumption good. \( W_t \) denotes the nominal wage, \( B_t \) represents the quantity of one-period nominally riskless discount bonds purchased in period \( t \) and maturing in period \( t+1 \). Each bond pays one unit of money at maturity and its price is \( Q_t \). \( T_t \) represents lump-sum additions or subtractions to period (i.e., lump-sum taxes, dividends, etc.), express in nominal terms. When solving the problem above, the household is assumed to take as given the price of the good, the wage, and the price of bonds.

In addition to (2), it is assume that the household is subject to a solvency constraint that prevents it from engaging in Ponzi-type schemes. The following constraint:
\[ \lim_{T \to \infty} E_t \{ B_T \} \geq 0; \quad \forall \ t \tag{3} \]

**Optimal Consumption and Labor Supply**

The Lagrangian is
\[ L = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \{ U(C_t, N_t) + \lambda_t (-P_t C_t - Q_t B_t + B_{t-1} + W_t N_t - T_t) \} \right] \]

and the first-order conditions are
\[ U_{c,t} - \lambda_t P_t = 0 \iff \lambda_t = \frac{U_{c,t}}{P_t} \tag{4} \]
\[ U_{n,t} + \lambda_t W_t = 0 \tag{5} \]
\[ \lambda_t (-Q_t) + E_t [\beta \lambda_{t+1}] = 0 \tag{6} \]

Combining equations (4) and (5) we have
\[ U_{n,t} + \frac{1}{P_t} U_{c,t} W_t = 0 \]
\[ -U_{N,t} = U_{c,t} \left( \frac{W_t}{P_t} \right) \tag{7} \]

Combining equations (4) and (6)
\[ \frac{Q_t}{P_t} U_{c,t} = E_t \left[ \beta \frac{U_{c,t+1}}{P_{t+1}} \right] \tag{8} \]
Interpretations

- **(4):** \( \lambda_t \) is the marginal utility of nominal income. If the household gets an extra dollar, he can purchase \( \left( \frac{1}{P_t} \right) \) additional units of the consumption good, while \( U_{c,t} \) is his marginal utility of consumption. Thus, \( \left( \frac{U_{c,t}}{P_t} \right) \) is the marginal benefit of nominal income, measured in utility. It is the additional utility the household receives by obtaining one more unit of income.

- **(7):** On the LHS, \(-U_{n,t}\) is the opportunity cost of working (i.e., the marginal disutility of labor). On the RHS, \( \left( \frac{W_t}{P_t} \right) \) is the additional units of the consumption good the household can purchase if he works an extra hour, while \( U_{c,t} \) is his marginal utility of consumption. Thus, the RHS shows the marginal benefit of working, measured in utility.

- **(8):** On the LHS, \( \left( \frac{Q_t}{P_t} \right) \) is the additional units of the consumption good the household can purchase if he sells a bond at time \( t \), while \( U_{c,t} \) is his marginal utility of consumption. Thus, the LHS is the marginal benefit you get by selling a bond in period \( t \), measured in utility. On the RHS, \( E_t \left\{ \beta \left( \frac{U_{c,t+1}}{P_{t+1}} \right) \right\} \) is the expected discounted marginal benefit of consumption in period \( t + 1 \), which is the additional utility the household receives if he keeps the bond until period \( t + 1 \).

Assumption on Utility

Much of what follows, assumes that the period utility takes the form

\[
U(C_t, N_t) = C_t^{1-\sigma} N_t^{1+\phi} \tag{9}
\]

Thus, the consumer’s optimality conditions (7) and (8) become

\[
N_t^{\phi} = C_t^{-\sigma} \left( \frac{W_t}{P_t} \right) \tag{10}
\]

\[
Q_t = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{11}
\]

Condition (10) can be interpreted as a competitive labor supply schedule, determining the quantity of labor supplied as a function of the real wage, given the marginal utility of consumption (which under the assumptions is a function of consumption only).

Log-Linearization

Note, for future reference, that equation (10) can be rewritten in log-linear form as

\[
w_t - p_t = \sigma C_t + \phi n_t \tag{12}
\]
where the lower case letters denote the natural logs of the corresponding variable (i.e., $x_t \equiv \log X_t$). We can also obtain a log-linear approximation of (11) around a steady state with constant rates of inflation and consumption growth

$$1 = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1} Q_t} \right]$$

1 = $E_t \{\exp [-\sigma(c_{t+1} - c_t) - \pi_{t+1} + i_t - \rho]\}$

where $\pi_{t+1} \equiv p_{t+1} - p_t$ is the inflation between $t$ and $t+1$, $i_t \equiv -\log Q_t$, and $\rho \equiv -\log \beta$ is the discount rate. We now construct the first-order Taylor approximation of the RHS around the zero inflation steady state, i.e., $(c_t, c_{t+1}, \pi_{t+1}, i_t) = (\bar{c}, \bar{c}, 0, \bar{\rho})$

$$1 = E_t \{1 + (-\sigma)(c_{t+1} - c) + \sigma(c_t - c) - \pi_{t+1} + (i_t - \rho)\}$$

$$0 = E_t \{-\sigma(c_{t+1} - c_t) - \pi_{t+1} + i_t - \rho\}$$

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - \rho)$$

Notice that $i_t$ corresponds to the log of the gross yield on the one-period bond; henceforth, it is referred to as the nominal interest rate. The yield on the one period bond is defined by $Q_t \equiv (1 + \text{yield})^{-1}$. Note that $i_t \equiv -\log Q_t = \log(1 + \text{yield}) \approx \text{yield_t}$, where the latter approximation will be accurate as long as the nominal yield is “small.” Similarly, $\rho$ can be interpreted as the household’s discount rate.

**Firms**

A representative firm is assumed whose technology is described by a production function given by

$$Y_t = A_t N_t^{1-\alpha}$$

where $A_t$ represents the level of technology, and $a_t \equiv \log A_t$ evolves exogenously according to some stochastic process.

Each period the firm maximizes profits

$$P_t Y_t - W_t N_t$$

subject to (11), taking the price and wage as given.

Maximization of (14) subject to (15) yields the optimality condition

$$P_t (1-\alpha) \frac{Y_t}{N_t} - W_t = 0$$

$$\frac{(1-\alpha)Y_t}{N_t} = \frac{W_t}{P_t}$$

(16)

Thus, optimal labor is obtained by the firm when the marginal product of labor (LHS) equals the real wage (RHS). Equivalently, the marginal cost $\frac{W_t}{(1-\alpha)A_t N_t^{-\alpha}}$ must be equated to the price $P_t$.  

Page 4 of 7
Log-Linearization

The log-linear aggregate production relationship is

\[ y_t = a_t + (1 - \alpha)n_t \]  

(17)

We can also rewrite the firm’s optimality condition in log-linear terms

\[
\log \left( \frac{W_t}{P_t} \right) = \log \left( \frac{(1 - \alpha) Y_t}{N_t} \right) \\
\log W_t - \log P_t = \log \left( (1 - \alpha)A_tN_t^{-\alpha} \right) \\
w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha) 
\]

(18)

Equation (18) can be interpreted as labor demand schedule, mapping the real wage into the quantity of labor demanded, give the level of technology.

Equilibrium

The baseline model abstracts from aggregate demand components like investment, government purchases, or net exports. Accordingly, the goods market clearing conditions is given by

\[ y_t = c_t \]  

(19)

Most importantly, the central bank chooses the nominal interest rate, \( i_t \).

We can summarize the model using the following equations:

\[ \omega_t = \sigma c_t + \phi n_t \]
\[ c_t = E_t \{ c_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho) \]
\[ \omega_t = a_t - \alpha n_t + \log(1 - \alpha) \]
\[ y_t = a_t + (1 - \alpha)n_t \]

This is a system of 4 equations and 4 endogenous variables \( (n_t, y_t, \omega_t, \pi_{t+1}) \) where \( \omega_t \equiv w_t - p_t \) is the real wage at time \( t \).

First, we combine the optimality conditions of households and firms with (19) and the log-linear aggregate production relationship and solve for \( n_t \).

\[
\sigma [a_t + (1 - \alpha)n_t] + \phi n_t = a_t - \alpha n_t + \log(1 - \alpha) \\
\sigma(1 - \alpha)n_t + \phi n_t + \alpha n_t = a_t - \sigma a_t + \log(1 - \alpha) \\
[\sigma(1 - \alpha) + \phi + \alpha] n_t = (1 - \sigma)a_t + \log(1 - \alpha) \\
\]

\[ n_t = \frac{1 - \sigma}{\sigma(1 - \alpha) + \phi + \alpha} a_t + \frac{\log(1 - \alpha)}{\sigma(1 - \alpha) + \phi + \alpha} \]

\[ n_t = \psi_{nt} a_t + \nu_n \]  

(20)
where
\[
\psi_{na} \equiv \frac{1 - \sigma}{\sigma(1 - \alpha) + \phi + \alpha} \quad \text{and} \quad \nu_n \equiv \frac{\log(1 - \alpha)}{\sigma(1 - \alpha) + \phi + \alpha}
\]

Equation (20) is the equilibrium level of employment as a function of the level of technology in the current period.

Next, we will analyze the firm’s production function
\[
y_t = a_t + (1 - \alpha) n_t
\]
\[
= a_t + (1 - \alpha) [\psi_{na} a_t + \nu_n]
\]
\[
= [1 + (1 - \alpha) \psi_{na}] a_t + (1 - \alpha) \nu_n
\]
\[
= \psi_{ya} a_t + \nu_y
\]
(21)

where
\[
\psi_{ya} \equiv \frac{1 + \phi}{\sigma(1 - \alpha) + \phi + \alpha} \quad \text{and} \quad \nu_y \equiv (1 - \alpha) \nu_n
\]

Equation (21) is the equilibrium level of output as a function of the level of technology in the current period.

Furthermore, given the equilibrium process for output, (13) can be used to determine the implied real interest rate, \(r_t \equiv i_t - E_t \{\pi_{t+1}\}\), as
\[
y_t = E_t \{y_{t+1}\} - \frac{1}{\sigma}(r_t - \rho)
\]
\[
\frac{1}{\sigma}(r_t - \rho) = E_t \{\Delta y_{t+1}\}
\]
\[
r_t = \rho + \sigma \psi_{ya} E_t [\Delta a_{t+1}]
\]
(22)

where \(\Delta y_{t+1} \equiv y_{t+1} - y_t\), \(\Delta a_{t+1} \equiv a_{t+1} - a_t\).

Finally, the equilibrium real wage \(\omega_t \equiv w_t - p_t\) is given by
\[
\omega_t = a_t - \alpha \psi_{na} a_t + \log(1 - \alpha)
\]
\[
= a_t - \alpha [\psi_{na} a_t + \nu_n] + \log(1 - \alpha)
\]
\[
= (1 - \alpha \psi_{na}) a_t - [\alpha \nu_n + \log(1 - \alpha)]
\]
\[
= \psi_{\omega a} a_t + \nu_{\omega}
\]
where
\[
\psi_{\omega a} \equiv \frac{\sigma + \phi}{\sigma(1 - \alpha) + \phi + \alpha} \quad \text{and} \quad \nu_{\omega} \equiv \frac{[\sigma(1 - \alpha) + \phi] \log(1 - \alpha)}{\sigma(1 - \alpha) + \phi + \alpha}
\]
Summary

Notice that the equilibrium dynamics of employment, output, the real interest rate, and the real wage rate are determined independently of monetary policy. In other words, monetary policy is neutral with respect to those real variables. In the simple model, output and employment fluctuate in response to variations in technology, which is assumed to be the only real driving force. It would be straightforward to introduce other real driving forces like variations in government purchases or exogenous shifts in preferences. In general, real variables will be affected by all those real shocks in equilibrium.

In particular, output always rises in the face of a productivity increase, with the size of the increase being given by \( \psi_{yw} > 0 \). The same is true for the real wage. On the other hand, the sign of the employment is ambiguous depending on whether \( \sigma \) (which measures the strength of the wealth effect of labor supply) is larger or smaller than one. When \( \sigma < 1 \), the substitution effect on labor supply resulting from a higher real wage dominates the negative effect caused by a smaller marginal utility of consumption, leading to an increase in employment. The converse is true whenever \( \sigma > 1 \). When the utility of consumption is logarithmic (\( \sigma = 1 \)), employment remains unchanged in the face of technology variations for substitution and wealth effects exactly cancel one another.

Next, the response of the real interest rate depends critically on the time series properties of technology. If the current improvement in technology is transitory so that \( E_t \{ a_{t+1} \} < a_t \), then the real rate will go down. Otherwise, if technology is expected to keep improving, then \( E_t \{ a_{t+1} \} > a_t \) and the real rate will increase with a rise in \( a_t \).

Finally, we consider nominal variables, like inflation or the nominal interest rate. Not surprisingly, and in contrast with real variables, their equilibrium behavior cannot be determined uniquely by real forces. Instead, it requires specification of how monetary policy is conducted.